DOCUMENT RESUME

ED 121 813 TM 005 242

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TITLE Regression Approaches and Approximate Solutions to Analysis of Variance with Disproportionality Varied.
PUB DATE 8 Jan 76
NOTE 18p.; Paper presented at the Annual Meeting of the American Educational Research Association (60th, San Francisco, California, April 19-23, 1976)

EDRS PRICE MF-$0.83 HC-$1.67 Plus Postage
DESCRIPTORS *Analysis of Variance; *Comparative Analysis; Data Analysis; Mathematical Models; Multiple Regression Analysis; *Statistical Analysis
IDENTIFIERS Data Elementary Method; Expected Frequencies Method; Least Squares Analysis; *Regression Solutions; Unweighted Means Analysis

ABSTRACT

The degree of nonorthogonality in a factorial design was systematically increased. Five methods of dealing with nonorthogonality were selected and applied: two were least squares solutions (Method 1 and Method 2); two were approximate solutions (the unweighted means analysis and the method of expected frequencies); and the fifth was the alternative of data elimination. Under extreme nonorthogonality all methods converged in yielding conclusions which while erroneous were similar across methods. Under moderate nonorthogonality, however, the unweighted means analysis and Method 1 were superior. Overall, the data elimination alternaitive was inferior in that it led to more type 2 errors than any of the other four methods. (Author)

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ED121813

Regression Approaches and Approximate Solutions to Analysis of Variance with Disproportionality Varied

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January 8, 1976

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TM005 242

Session 13.25

Abstract

The degree of nonorthogonality in a factorial design was systematically increased. Five methods of dealing with nonorthogonality were selected and applied: two were least squares solutions (Method 1 and Method 2); two were approximate solutions (the unweighted means analysis and the method of expected frequencies); and the fifth was the alternative of data elimination. Under extreme nonorthogonality all methods converged in yielding conclusions which while erroneous were similar across methods. Under moderate nonorthogonality, however, the unweighted means analysis and Method 1 were superior. Overall, the data elimination alternative was inferior in that it led to more type II errors than any of the other four methods.

Regression Approaches and Approximate Solutions to Analysis of Variance with Disproportionality Varied

Authors of current textbooks in design and statistics (Keppel, 1973; Dayton, 1970) acknowledge the superiority of the least squares methods yet refer researchers whose data evidences only slight departures from proportionality to the more easily understood and computationally simpler approximate solutions. The most commonly recommended ones are the unweighted means analysis and the method of expected frequencies. Data elimination to achieve equal n is also frequently recommended.

Several investigators, (particularly Carlson & Timm, 1974; Timm & Carlson, 1975; Overall, Spiegel, & Cohen, 1975; Cohen & Cohen, 1975; & Overall & Spiegel, 1969) have compared the various regression solutions and clarified the hypotheses tested by each. Yet, despite this clarification no one has empirically compared the best known regression solutions to the more popular approximate ones.

Three regression procedures for analyzing disproportional data will be defined. The reader who is not familiar with these methods is referred to the seminal article by Overall and Spiegel and a more recent investigation by Carlson and Timm. These solutions are conceptually and algebraically similar in the univariate and the multivariate cases (Woodward & Overall, 1975), but the present discussion will be limited to univariate analyses since multivariate extensions of the approximate solutions have not been developed.

Method 1 requires mutual adjustment of all main effects and interactions. The structural model for a two-way analysis of variance is: $X_{ijm} = \mu + \alpha_i + \beta_j + \alpha\beta_{ij} + e_{ijm}$, where μ is the grand mean, α_i is the treatment effect for level i of the first factor, β_j is the treatment effect for level j of the second factor, $\alpha\beta_{ij}$ is the interaction term for cell ij, and e_{ijm} is the error for individual m in cell ij. Carlson and Timm have argued cogently that Method 1 is the best extension of traditional analysis of variance because the same parameters are estimated and the same hypotheses are tested in the orthogonal and the nonorthogonal cases. No rigorous proof is known, but empirical tests carried out by the present author indicate that results with Method 1 coincide with results found by using the weighted squares of means method as described by Winer (1971) and by Schuessler (1971).

Method 2 is similar to Method 1. The structural model for Method 2 is $X_{ijm} = \mu + \alpha_i + \beta_j + e_{ijm}$ where the terms are defined as they were for Method 1. The essential difference between the two methods is that Method 2 requires the assumption that no true interaction exists and consequently the interaction is not taken into consideration when estimating main effects. An empirical test shows that the results with Method 2 are identical to those with the method of fitting constants as described by Winer. In fact, the congruence of these two methods is so obvious it seems odd that they are still referred to by separate names. Based primarily on tradition and popular acceptance, Overall and Spiegel initially suggested that Method 2 was the best generalization of analysis of variance to the

nonorthogonal case. However, they later reversed their stance (Overall, Spiegel, & Cohen) and argued that Method 1 is the proper generalization of analysis of variance to the nonorthogonal case. Methods 1 and 2 are identical when the interaction is exactly zero.

The structural model for Method 3 is identical to the model for Method 1. Method 3, however, assumes a priori evidence to justify an ordered entry of vector sets representing α, β, and $\alpha\beta$ into the regression equation. The hypotheses tested with Method 3 are not the same as those tested with analysis of variance. While this method has the advantage that the component sums of squares equal the total sum of squares, the requirement of establishing a logical a priori ordering of variables is typically impossible for a researcher to establish, thus it is of very limited usefulness. Consequently it will not be considered in this study.

The approximate solutions for the analysis of nonorthogonal data are adequately described in a number of textbooks (Myers, 1967; Winer, 1971; Kirk, 1968; & Glass & Stanley, 1970). The unweighted means analysis uses cell means to estimate main effects and interaction, and adjusts the error term by a factor which reflects the unequal cell sizes. The method of expected frequencies involves multiplication of cell sums by the expected cell frequency to obtain a sum for each cell. Sums obtained in this manner are used in estimating main effects and interactions. The third and final commonly recommended non least squares procedure to be considered here is not really a solution to nonorthogonal data analysis: it consists of randomly eliminating data to achieve equal n.

Some general, but largely unsubstantiated, guidelines are available concerning the appropriate selection of a least squares or an approximate solution. Winer and Dayton indicate that the unweighted means analysis is applicable if the experimental design called for equal n and if subject loss was essentially random. Myers (1967) offers a similar caution. In addition, he warns the experimenter not to use the unweighted means analysis when the "n's are very disparate." Myers also views the method of expected frequencies as appropriate when proportionality can be assumed and when departure from proportionality is not too great. When the departure is large, he indicates that a least squares method should be undertaken. If one follows this recommendation, a least squares solution would be selected when disproportionality is relatively large.

Williams (1972) has compared certain selected least squares and non least squares methods. However, he omitted the least squares solution (Method 1) which in view of the subsequent research of Carlson and Timm may well be the most acceptable generalization of traditional analysis of variance to the nonorthogonal case. In addition, Williams included a discussion of the hierarchical method (Method 3), the unadjusted main effects method, and the method of weighted means. These three methods are of limited utility. The difficulty inherent in the hierarchical method has been discussed above. The method of weighted means is seldom recommended when there are two or more missing scores per cell. The unadjusted main effects method places the variance shared by main effects into all main effect estimates thereby unjustifiably inflating the F values and leading to type I errors.

No one seems to have indicated when departures from proportionality should be considered large or to have established a criterion for judging when a least squares solution becomes more appropriate than a non least squares procedure. The main purpose of this study is determine if the least squares solutions and approximate solutions diverge as the degree of nonorthogonality increases. Based on the literature review it is hypothesized that they will diverge when nonorthogonality is extreme.

Other possible methods were also considered but omitted from detailed examination due to characteristics which mark them as clearly unacceptable. These were (1) a modification of Method 2 which entails pooling insignificant interactions and the error term, and (2) substitution of the cell mean for the missing values in that cell. The mean substitution method tends to yield F values much higher than those in the basic data analysis which is conducted before any subject loss is incurred. A clearly unacceptable rate of type II errors ensues. The pooling procedure in conjunction with Method 2, even when the interaction is insignificant, can have effects which are very difficult to predict because the pooled interaction will itself change with increasing degrees of nonorthogonality. As a result it was not included in the detailed comparisons which follow.

Method

Two 3 X 3 factorial designs were used as the initial data sets (Table 1). Each basic data set contains 81 scores, nine per cell, but only three

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Insert Table 1 about here

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different scores. Such score duplication, which allows frequency reduction and the introduction of nonorthogonality without modifying cell means or the parameter estimated was used by Overall, Spiegel, and Cohen. Analysis of Basic Data I results in a significant interaction as well as two significant main effects. Basic Data II differs in that it produces no significant interaction. Analyses of Basic Data I and II are assumed to represent the correct results.

Case 1 and Case 3 represent reductions in frequencies that lead to moderate (Case 1) and extreme (Case 3) nonorthogonality. Further reduction of the frequencies found in Case 1 and in Case 3 leads to the equal, although reduced, frequencies found in Case 2 and Case 4. The four solutions, i.e., Method 1, Method 2, the method of expected frequencies, and the unweighted means analysis, will be applied to both sets of data under both conditions of nonorthogonality. Based on these results the four solutions will be compared to each other and to the fifth alternative of further eliminating subjects to obtain frequencies in all cells. Such elimination produced Case 2 and Case 4.

An overview of the various analyses and comparisons can be offered in terms of Table 2. The data in cells G and H will be analyzed using the four

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Insert Table 2 about here

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solutions to nonorthogonal data analysis and these results will be compared to the results from analysis of data in cells A and B. The four solutions to the nonorthogonal data in cells G and H will also be compared to the analyses

of the data configurations in cells C and D. The data in cells I and J, representing extreme nonorthogonality, will be analyzed using all four solutions and the results will be compared to the results from the analysis of data in cells E and F. Results from cells I and J will also be compared to results from cells E and F. As can be seen in Table 2, it is possible to compare the four solutions to each other and to the alternative of eliminating scores to achieve equal n both when nonorthogonality is moderate and when it is extreme.

Results and Discussion

When nonorthogonality was moderate and when no interaction was present (Table 3), Method 1 and the unweighted means analysis both yielded

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Insert Table 3 about here

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essentially the same conclusions ($\alpha = .05$) that were reached by analysis of Basic Data I prior to any frequency reduction whatsoever, namely, rejection of all three null hypotheses: $\Sigma\alpha_i^2 = 0$, $\Sigma\beta_j^2 = 0$, and $\Sigma(\alpha_i\beta_j)^2 = 0$. Method 2, the method of expected frequencies, and further data reduction to achieve equal n (Case 2) would all lead to the same error, the failure to reject $\Sigma\beta_j^2 = 0$.

When nonorthogonality was extreme and no interaction was present (Case 3) all four solutions and the alternative of further data elimination (Case 4) led to essentially the same results. Thus, contrary to what the recommendations of Winer and Myers might lead one to anticipate, Method 1 did not become more preferable as deviations from orthogonality increased.

To the contrary, when nonorthogonality was extreme the F values resulting from application of Method 1 deviate more from the standard than do the F values resulting from either of the approximate solutions. Method 2 was not an appropriate analysis because a significant interaction was present in Basic Data I.

The analysis of Basic Data II was used as a standard for comparing the various methods of dealing with nonorthogonality when no interaction is present (Table 4). When nonorthogonality was moderate all four solutions led to the same results as those found in the standard, namely re-

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Insert Table 4 about here

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jection of $\Sigma\alpha_i^2 = 0$, and $\Sigma\beta_j^2 = 0$, but failure to reject $\Sigma(\alpha_i\beta_j)^2 = 0$. There was a type II error in Case 2, the data elimination alternative. It consisted of the failure to reject $\Sigma\beta_j^2 = 0$.

When no interaction was present and nonorthogonality was extreme (Case 3) all four solutions and the data elimination alternative led to the same conclusions and resulted in one type II error. Again it is found that the conclusions reached by least squares and those reached by approximate methods do not diverge with increased nonorthogonality.

Summary

In comparing the four solutions, (a) to the analyses of the initial factorials, (b) to each other, and (c) to the data elimination alternative, it was obvious that, regardless of the degree of nonorthogonality and regardless of the presence or absence of an interaction, data elimination

to achieve equal n was the worst alternative due to the strong tendency to yield type II errors.

When nonorthogonality was extreme, contrary to the hypothesis put forth in this paper, the four solutions tended to converge in yielding the same results. A slight divergence of results was found when a moderate degree of nonorthogonality was present, but not along the dimension of regression solutions versus nonregression solutions. Rather Method 1 and the unweighted means analysis appear to be best when results differ.

Method 1 is probably the most defensible based on criteria external to this study (Carlson and Timm); yet the present results indicate that the more traditional solutions yield F values very close to the F values obtained by Method 1. In general, it appears that the traditional solutions may serve as satisfactory substitutes for Method 1 when interaction is present or for Method 1 or Method 2 when no interaction is present.

These conclusions are important to researchers who are accustomed to the unweighted means analysis or the method of expected frequencies but feel the more computationally complex regression solutions should be adopted. In many cases this fear may be unfounded. The present results are also important to those who use the regression approaches to nonorthogonal data analyses. It offers some assurance that results obtained via regression analyses are comparable to analyses from previous studies conducted by researchers who used the method of expected frequencies or the unweighted means analysis when they encountered disproportional data.

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Table 1

Two sets of basic data and four cases in which certain cell frequencies ( ) were reduced to simulate random data loss or to achieve equal n.

Basic Data I
Interaction Present

3 6(9) 9	6 4(9) 7	9 12(9) 14
5 6(9) 8	6 5(9) 7	3 5(9) 8
10 12(9) 9	11 13(9) 15	12 8(9) 10

Basic Data II
No Interaction Present

3 5(9) 8	3 4(9) 9	10 6(9) 9
6 6(9) 7	5 6(9) 8	9 12(9) 8
5 12(9) 7	4 14(9) 10	11 13(9) 15

(9)	(6)	(6)
(6)	(9)	(6)
(6)	(9)	(9)

1. Case 1, moderate nonorthogonality. $\chi^2 = 2.2$, N = 66

(6)	(6)	(6)
(6)	(6)	(6)
(6)	(6)	(6)

2. Case 2. Frequencies reduced from 66 found in Case 1 to 54 to achieve equal n.

(9)	(3)	(3)
(3)	(9)	(3)
(3)	(9)	(9)

3. Case 3, extreme nonorthogonality $\chi^2 = 11.99$, N = 51

(3)	(3)	(3)
(3)	(3)	(3)
(3)	(3)	(3)

4. Case 4. Frequencies reduced from 51 found in Case 3 to 27 to achieve equal n.

Table 2

Schematatic representations of all data configurations and two sets of data.

Data Configurations	Basic Data I	Basic Data II
Equal n (n = 9)	A	B
Case 2 (n = 6)	C	D
Case 4 (n = 3)	E	F
Case 1, moderate nonorthogonality (χ^2 = 2.20)	G	H
Case 3, extreme nonorthogonality (χ^2 = 11.99)	I	J

Table 3

Data analyses of Basic Data I (Interaction present) with varying degrees of nonorthogonality.

Degree of Orthogonality		Basic Data I Results used as a standard for comparison			Case 2			Case 4		
		df	F	P	df	F	P	df	F	P
Orthogonal Data (N=81, 54, 27)	A	2	60.12	.0001	2	37.58	.0001	2	15.04	.0001
	B	2	4.49	.0145	2	2.81	.0708	2	1.12	.3480
	AB	4	18.30	.0001	4	11.44	.0001	4	4.58	.0100
	WG	72			45			18		

Four Solutions to Nonorthogonal Data Analysis

		Method 1			Method 2		Expected Frequencies Method		Unweighted Means Analysis	
		df	F	P	F	P	F	P	F	P
Case 1 Moderate Nonorthogonality (χ^2=2.2, N=66)	A	2	46.66	.0001	48.97	.0001	50.56	.0001	45.12	.0001
	B	2	3.23	.0469	3.06	.0546	3.01	.0572	3.38	.0408
	AB	4	13.31	.0001	13.31	.0001	14.01	.0001	13.73	.0001
	WG	57								
		df	F	P	F	P	F	P	F	P
Case 3 Extreme Nonorthogonality (χ^2=11.99, N=51)	A	2	27.79	.0001	37.49	.0001	40.36	.0001	25.50	.0001
	B	2	1.72	.1914	2.57	.0885	1.77	.1828	1.91	.1601
	AB	4	7.64	.0001	7.64	.0001	9.85	.0001	7.76	.0001
	WG	42								

Table 4

Data analyses of Basic Data II (no interaction present) with varying degrees of nonorthogonality.

Degree of Orthogonality		Basic Data II Results used as a standard for comparison			Case 2			Case 4		
		df	F	P	df	F	P	df	F	P
Orthogonal Data (N=81, 54, 27)	A	2	79.01	.0001	2	49.38	.0001	2	19.75	.0001
	B	2	4.58	.0131	2	2.87	.0671	2	1.15	.3388
	AB	4	.68	.6081	2	.43	.7862	4	.17	.9509
	WG	72			45			18		

Four Solutions to Nonorthogonal Data Analysis

			Method 1		Method 2		Expected Frequencies Method		Unweighted Means Analysis	
		df	F	P	F	P	F	P	F	P
Case 1 Moderate Nonorthogonality (χ^2=2.2, N=66)	A	2	56.75	.0001	59.18	.0001	59.60	.0001	58.47	.0001
	B	2	3.50	.0368	3.63	.0328	3.59	.0339	3.40	.0403
	AB	4	.50	.7358	.50	.7358	.56	.6930	.50	.7358
	WG	57								
Case 3 Extreme Nonorthogonality (χ^2=11.99, N=51)	A	2	30.26	.0001	36.06	.0001	40.64	.0001	32.37	.0001
	B	2	2.03	.1440	2.38	.1049	2.59	.0873	1.88	.1652
	AB	4	.28	.8893	.28	.8893	.34	.8498	.28	.8893
	WG	42								